

# Indications for Large Rescattering in Rare B Decays

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**Abstract.** The sign of  $A_{CP}(K^-\pi^+) < 0$ , the evidence for  $\bar{B}^0 \rightarrow \pi^0\pi^0$ , and the possibly sizable  $A_{\pi\pi}$  and  $S_{\pi\pi}$  in  $\bar{B}^0 \rightarrow \pi^+\pi^-$  all suggest that final state rescattering may be needed in  $\bar{B} \rightarrow PP$  decay, which is echoed by large color suppressed  $\bar{B}^0 \rightarrow D^0h^0$  modes. An SU(3) formalism of  $\mathbf{8} \otimes \mathbf{8} \rightarrow \mathbf{8} \otimes \mathbf{8}$  rescattering in  $PP$  final states leads to interesting predictions, in particular allowing for small  $\bar{B}^0 \rightarrow K^-K^+$ .

**PACS.** 11.30.Hv Flavor symmetries – 13.25.Hw Decays of bottom mesons

## 1 Motivation

Around 1999, the emergence of large  $K\pi/\pi\pi$  ratio in  $B$  decay lead to the suggestion [1] that maybe  $\gamma \equiv \phi_3 \equiv \arg V_{ub}^* \gtrsim 90^\circ$ , in contrast to the CKM fit (to other data) of  $\sim 60^\circ$ . The pattern of  $K\pi$ ,  $\pi\pi$  data can then be understood within factorization. As the final results from CLEO came out, it was further speculated [2] that rate and (direct)  $A_{CP}$  pattern could hint at rescattering in final state (FSI). If one makes a final state isospin decomposition, FSI phases are *in principle* present. As  $A_{CP}$ s depend critically on absorptive parts, the CP invariant FSI phases could easily shift direct CP patterns.

The discovery of color suppressed  $\bar{B}^0 \rightarrow D^0h^0$  decays [3] above factorization predictions suggests that FSI may have to be taken seriously. More recently [4], the  $3\sigma$  effect of  $A_{CP}(K^-\pi^+) < 0$ , the evidence for  $\bar{B}^0 \rightarrow \pi^0\pi^0$ , and the possibly sizable  $A_{\pi\pi}$  and  $S_{\pi\pi}$  in  $\bar{B}^0 \rightarrow \pi^+\pi^-$  etc., all could be hinting at presence of sizable rescattering in  $\bar{B} \rightarrow PP$  final states, where  $P$  stands for an octet pseudoscalar. We thereby revisit the FSI speculation.

Treating the color suppressed  $\bar{B}^0 \rightarrow D^0h^0$  modes as an exercise, we developed [5] an SU(3) based  $\mathbf{3} \otimes \mathbf{8} \rightarrow \mathbf{3} \otimes \mathbf{8}$  rescattering in  $DP$  final states. Here we report the results [6] on extending the formalism to  $PP$  final states.

## 2 Ansatz: Multimode Fit with FSI Phases

Since factorization seems to account for the rates of leading decays, we adopt the simple and physical picture of (naively) factorized amplitudes  $\mathcal{A}_i^f$  followed by FSI, i.e.

$$\langle i; \text{out} | H_W | B \rangle = \sum_i \mathcal{S}_{ii}^{1/2} \mathcal{A}_i^f. \quad (1)$$

We use naive factorization not just for sake of simplicity, but because more sophisticated treatment in, say, QCD

factorization introduces hadronic parameters, and one may incur double counting. Note that  $l$  is summed over quasi-elastic channels in Eq. (1). We assume that the large cancellations between numerous *inelastic* channels generate only the “perturbative” FSI phase accounted for by the penguin absorptive part.

We treat  $\bar{B} \rightarrow PP$  final states only, since  $VP$  modes are not yet settled (both experiment and theory). Also, we are yet unable to treat  $\eta'$  hence take  $\eta \cong \eta_8$ . Thus, we consider  $\mathbf{8} \otimes \mathbf{8} \rightarrow \mathbf{8} \otimes \mathbf{8}$  rescattering. Since only the  $\mathbf{1}$ , one of the  $\mathbf{8}$ s, and the  $\mathbf{27}$  are symmetric, the  $\mathcal{S}^{1/2}$  matrix in Eq. (1) takes up the form

$$\mathcal{S}^{1/2} = e^{i\delta_{27}} |\mathbf{27}\rangle \langle \mathbf{27}| + e^{i\delta_8} |\mathbf{8}\rangle \langle \mathbf{8}| + e^{i\delta_1} |\mathbf{1}\rangle \langle \mathbf{1}|, \quad (2)$$

hence there are just two physical phase differences, which we take as  $\delta \equiv \delta_{27} - \delta_8$  and  $\sigma \equiv \delta_{27} - \delta_1$ . These FSI phases redistribute  $\mathcal{A}_i^f$  according to Eq. (1). Alternatively, they can be viewed as a *simple two parameter model* extension beyond the usual  $B \rightarrow PP$  amplitudes.

It is important to point out that the  $\sigma$  phase appears only in the  $\pi^-\pi^+$ ,  $\pi^0\pi^0$ ,  $K^-K^+$ ,  $K^0\bar{K}^0$ ,  $\pi^0\eta_8$  and  $\eta_8\eta_8$  rescattering subset. It arises from a total (or double) annihilation of the incoming  $PP$  state, i.e.  $(q_1\bar{q}_2)(\bar{q}_1q_2) \rightarrow (q'_1\bar{q}'_2)(\bar{q}'_1q'_2)$ . Heuristically, one has three other type of “topologies”: “pomeron”, with exchange of only energy-momentum; “exchange”, where a (anti-)quark pair is exchanged; “annihilation”, where a quark-antiquark pair is annihilated. In the end, besides an overall magnitude and phase, one is left with two phase differences.

By adding the two strong phases  $\delta$  and  $\sigma$ , we follow the multimode fit approach of Ref. [7]. We take only the better measured or known quantities as input: 7 rates from  $K\pi$ ,  $\pi\pi$ , 3 asymmetries from  $K^-\pi^+$ ,  $K^-\pi^0$  and  $\bar{K}^0\pi^+$ , and the (theoretical) form factor ratio  $F_0^{BK}/F_0^{B\pi} = (0.9 \pm 0.1) f_K/f_\pi$ . The fit parameters are  $F_0^{BK}$ ,  $1/m_s^{\text{eff}}$ ,  $\delta$  and  $\sigma$ , and possibly  $\phi_3$ , where we explore the two cases of keeping it free (Fit 1), or fixed (Fit 2) at  $60^\circ$ . The fit output are the

**Table 1.** World average inputs and fitted outputs; data in brackets are not used in fit, while  $\eta_8 K(\pi)^-$  entries are for  $\eta K(\pi)^-$ . Horizontal lines separate rescattering subsets. Fit 1 or 2 stand for  $\phi_3$  free or fixed at  $60^\circ$ . Setting  $\delta = \sigma = 0$  but keeping other parameters fixed give the results in parentheses; the fitted parameters and  $\chi^2_{\min.}$  are given in Table 2.

Modes	$\mathcal{B}^{\text{expt}} \times 10^6$	$\mathcal{B}^{\text{Fit1}} \times 10^6$	$\mathcal{B}^{\text{Fit2}} \times 10^6$	$A_{\text{CP}}^{\text{expt}} (\%)$	$A_{\text{CP}}^{\text{Fit1}} (\%)$	$A_{\text{CP}}^{\text{Fit2}} (\%)$
$K^-\pi^+$	$18.2 \pm 0.8$	$19.4^{+1.0}_{-1.2} (19.7)$	$18.5 \pm 0.6 (18.1)$	$-9 \pm 4$	$-6^{+2}_{-3} (9)$	$-4 \pm 1 (7)$
$\bar{K}^0\pi^0$	$11.2 \pm 1.4$	$8.3^{+1.3}_{-0.4} (7.3)$	$9.1 \pm 0.3 (8.7)$	$[3 \pm 37]$	$24^{+4}_{-30} (0)$	$16^{+2}_{-21} (0)$
$\bar{K}^0\eta_8$	$[< 4.6 (90\% \text{ CL})]$	$3.4^{+0.8}_{-0.6} (4.1)$	$3.9^{+1.0}_{-0.8} (4.6)$	—	$24^{+9}_{-4} (0)$	$15^{+3}_{-2} (0)$
$K^0\pi^-$	$20.6 \pm 1.3$	$19.6^{+2.2}_{-1.4} (18.5)$	$21.6 \pm 0.6 (20.9)$	$1 \pm 6$	$8^{+2}_{-1} (0)$	$5 \pm 0 (0)$
$K^-\pi^0$	$12.8 \pm 1.1$	$11.6^{+0.5}_{-1.0} (12.1)$	$11.0 \pm 0.3 (10.9)$	$1 \pm 12$	$-19^{+4}_{-7} (7)$	$-14^{+1}_{-2} (6)$
$K^-\eta_8$	$[3.2 \pm 0.7]$	$3.6^{+0.8}_{-0.7} (4.2)$	$4.6^{+1.0}_{-0.8} (5.4)$	$[-32 \pm 20]$	$33^{+15}_{-9} (-9)$	$19^{+6}_{-4} (-5)$
$\pi^-\pi^0$	$5.3 \pm 0.6$	$4.4^{+1.2}_{-0.6} (4.4)$	$3.2^{+0.1}_{-0.2} (3.2)$	$[-7 \pm 14]$	$0 (0)$	$0 (0)$
$\pi^-\eta_8$	$[3.9 \pm 0.8]$	$1.2^{+0.1}_{-0.3} (1.4)$	$1.5^{+0.0}_{-0.1} (1.8)$	$[-51 \pm 19]$	$75^{+25}_{-18} (-32)$	$42^{+9}_{-6} (-19)$
$K^-K^0$	$< 2.2 (90\% \text{ CL})$	$1.7^{+0.3}_{-0.2} (1.5)$	$1.3 \pm 0.1 (1.0)$	—	$-84^{+9}_{-14} (-4)$	$-79 \pm 3 (-3)$
$\pi^-\pi^+$	$4.5 \pm 0.4$	$4.7^{+0.7}_{-0.8} (7.4)$	$5.1^{+0.3}_{-0.4} (8.7)$	$[51 \pm 23]$ $[-49 \pm 61]$	$A_{\pi\pi} = 12^{+14}_{-50} (-24)$ $S_{\pi\pi} = -15^{+69}_{-9} (-5)$	$13 \pm 2 (-17)$ $-90^{+1}_{-0} (-88)$
$\pi^0\pi^0$	$1.7 \pm 0.6$	$2.5^{+0.4}_{-0.9} (0.2)$	$2.8^{+0.3}_{-0.7} (0.1)$	—	$-56^{+1}_{-16} (1)$	$-35^{+1}_{-6} (1)$
$K^-K^+$	$< 0.6 (90\% \text{ CL})$	$0.2^{+0.4}_{-0.2} (0.0)$	$0.6^{+0}_{-0.1} (0.0)$	—	$-13^{+9}_{-76} (-)$	$-11^{+2}_{-1} (-)$
$\bar{K}^0K^0$	$< 1.6 (90\% \text{ CL})$	$1.5^{+0.3}_{-0.6} (1.5)$	$1.1 \pm 0.1 (1.0)$	—	$-63^{+141}_{-24} (-4)$	$-86^{+6}_{-1} (-3)$
$\pi^0\eta_8$	—	$0.4^{+0.2}_{-0} (0.3)$	$0.2 \pm 0.0 (0.2)$	—	$-3 \pm 0 (-4)$	$-3 \pm 0 (-4)$
$\eta_8\eta_8$	—	$0.2 \pm 0.0 (0.1)$	$0.2 \pm 0.1 (0.1)$	—	$-10^{+96}_{-75} (-6)$	$-91^{+14}_{-3} (-6)$

rates,  $A_{\text{CP}}$ s, and especially  $A_{\pi\pi}$  and  $S_{\pi\pi}$ . These, together with the inputs, are given in Table 1.

### 3 Results

In Table 1, the numbers given in parentheses are by setting  $\delta$  and  $\sigma$  to zero but keeping all other parameters as determined by the fit, which indicates the amount of FSI cross-feed. The  $\chi^2_{\min.}$  and fitted parameters are given in Table 2. For illustration, we obtain output errors for both Tables by scanning the  $\chi^2 \leq \chi^2_{\min.} + 1$  parameter space. The fitted rates and CP asymmetries, as well as the fitted parameters, when compared with data or with theory, gives one a sense of reasonableness. For example, the  $\chi^2_{\min.}/\text{d.o.f.}$  for Fit 1 and 2 are 17/5 (giving  $\phi_3 \cong 96^\circ$ ) vs. 25/6, and the former seems better. This can also be seen from the low  $F_0^{BK(\pi)}$  and  $m_s^{\text{eff}}$  fitted parameters from Fit 2, when compared with theory. Both fits are much worse without

**Table 2.** The  $\chi^2_{\min.}$  and fitted parameters for Fits 1, 2. We constrain  $f_\pi F_0^{BK}/f_K F_0^{B\pi} = 0.9 \pm 0.1$ , and  $1/m_s^{\text{eff}}$  gives the effective chiral enhancement for  $\langle O_6 \rangle$ . The last column is for  $\phi_3$  free (fixed) without FSI phases.

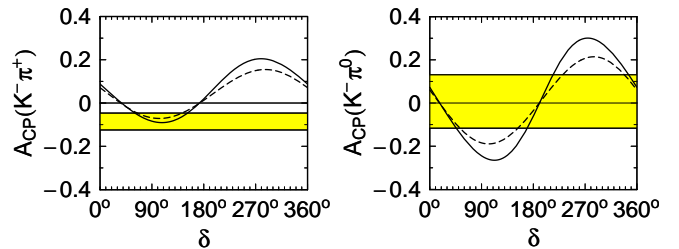
	Fit 1	Fit 2	No FSI
$\chi^2_{\min.}/\text{d.o.f.}$	17/5	25/6	50/7 (65/8)
$\phi_3$	$(96^{+21}_{-10})^\circ$	$[60^\circ]$	$106^\circ ([60^\circ])$
$\delta$	$(67^{+21}_{-11})^\circ$	$(63^{+6}_{-0})^\circ$	—
$\sigma$	$(90^{+14}_{-59})^\circ$	$(103^{+4}_{-0})^\circ$	—
$F_0^{B\pi}$	$0.29^{+0.04}_{-0.02}$	$0.24^{+0.00}_{-0.01}$	0.25 (0.16)
$F_0^{BK}$	$0.33^{+0.05}_{-0.06}$	$0.27^{+0.00}_{-0.02}$	0.27 (0.18)
$m_s^{\text{eff}}$ (MeV)	$81^{+26}_{-14}$	$57^{+1}_{-3}$	66 (35)

FSI:  $\chi^2_{\min.}/\text{d.o.f.}$  is 50/7 (65/8) for Fit 1 (2), as seen in the last column of Table 2.

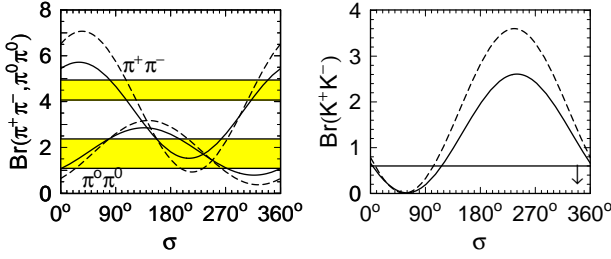
From Table 2 we see that, whether one keeps  $\phi_3$  fixed or free, the fitted FSI phases  $\delta$  and  $\sigma$  are rather sizable, while disallowing them gives much poorer  $\chi^2$ . Let's see what drives these phases.

The  $\delta$  dependence of  $A_{\text{CP}}$ s for  $K^-\pi^+$  and  $K^-\pi^0$  are plotted in Fig. 1. The former now has some significance, but *opposite in sign* w.r.t. QCD factorization predictions. We see that FSI can bring about a sign change, which disfavors  $\sin \delta < 0$ . Together with “restraint” from  $K^-\pi^0$  mode,  $\delta \sim 60^\circ$  is more or less settled between the two. Note that  $A_{\text{CP}}(K^-\pi^0) < -14\%$  from both fits. This is in contrast with  $(1 \pm 12)\%$  from current data, which averages out a sizable *positive* value of  $(23 \pm 11^{+1}_{-4})\%$  from Belle against *negative* central values reported by both BaBar and CLEO. From a theory standpoint,  $A_{\text{CP}}(K^-\pi^0)$  should basically track  $A_{\text{CP}}(K^-\pi^+)$ , as can be seen from fitted output, which should be tested with more data.

The  $\pi^-\pi^+$  and  $\pi^0\pi^0$  rates are sensitive to both  $\delta$  and  $\sigma$ . They are plotted in Fig. 2(a) with  $\delta$  fixed to fit values of



**Fig. 1.**  $A_{\text{CP}}(K^-\pi^{+,0})$  vs.  $\delta$ . Solid (dashed) line is for Fit 1 (Fit 2), and shaded bands are  $1\sigma$  experimental ranges.



**Fig. 2.** Rates ( $\times 10^6$ ) for (a)  $\pi^+\pi^-$ ,  $\pi^0\pi^0$ , (b)  $K^-K^+$  vs.  $\sigma$ . Solid (dashed) line is for Fit 1 (Fit 2) with  $\delta$  fixed at  $67^\circ$  ( $63^\circ$ ). Horizontal bands are  $1\sigma$  experimental ranges or upper limit.

Table 2. Both fits clearly favor large  $\sigma$ , to account for the smallness of  $\pi^-\pi^+$  rate by rescattering into  $\pi^0\pi^0$ , which both [8] BaBar and Belle now have evidence for!

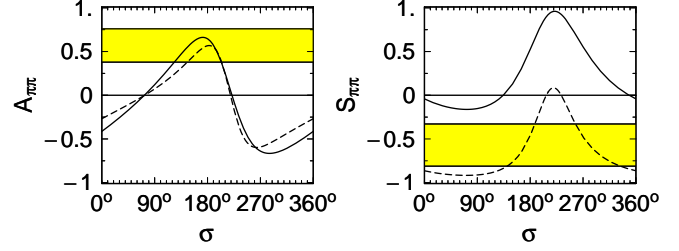
The rate of  $K^-K^+ < 6 \times 10^{-7}$  is very suppressed, which could challenge FSI. From Eq. (2), if  $\delta_1, \delta_8, \delta_{27}$  are all randomly sizable,  $K^-K^+ \sim \pi^-\pi^+ > 10^{-6}$  would be expected. However, as seen in Fig. 2(b), for  $|\delta - \sigma| \lesssim 50^\circ$ , the  $K^-K^+$  rate can be comfortably below the present limit, but  $\delta, \sigma$  can be separately large. The reason is due to the smallness of **27** in the  $I = 0$   $\pi\pi \rightarrow \pi\pi$  amplitude.  $K^0\bar{K}^0 \sim K^0K^- \sim 10^{-6}$  are, however, little perturbed.

Aside from the recent evidence for  $\pi^0\pi^0$ , the focus of our interest is the mixing-dependent CP violation in the  $\pi^+\pi^-$  mode, where Belle and BaBar have been reporting conflicting results since 2002. It should be noted that BaBar's  $113 \text{ fb}^{-1}$  update has moved closer to Belle's, which has yet to update but published results lie outside the physical region. Both experiments now agree in the signs, and the new summer 2003 averages are  $S_{\pi\pi} = -0.58 \pm 0.20$  and  $A_{\pi\pi} = 0.38 \pm 0.16$ . In any case, we have not used these numbers in our fit.

The  $\sigma$  dependence for  $A_{\pi\pi}$  and  $S_{\pi\pi}$  are plotted in Fig. 3 with  $\delta$  fixed to fit values of Table 2. As  $A_{\pi\pi}$  is nothing but  $A_{\text{CP}}(\pi^+\pi^-)$ , it has strong FSI dependence. Much like the case of  $K^-\pi^+$ , large  $\sigma \sim 100^\circ$  turns  $A_{\pi\pi}$  positive, of order 10%, in agreement with BaBar's update number  $-0.19 \pm 0.19 \pm 0.05$ .  $S_{\pi\pi}$  depends very weakly on  $\sigma$  for  $\sin \sigma > 0$ , the domain of interest. We have further checked that it is basically flat in the first quadrant of  $\delta$ - $\sigma$  plane. But, as a measure of indirect CP violation, it depends very strongly on the CP phase  $\phi_3$ . From Fig. 3(b) we see that it changes from  $\sim 0$  for  $\phi_3 \sim 96^\circ$  (Fit 1), and turns almost maximal negative ( $-90\%$ ) for  $\phi_3$  fixed to  $60^\circ$  (Fit 2). The situation is clearly as volatile as the actual competition between Belle and BaBar!

## 4 Discussion

The situation for  $A_{\pi\pi}$  and  $S_{\pi\pi}$  is indeed volatile, as evidenced by the recent shift of BaBar numbers closer to Belle's, while Belle has yet to update with full 2003 dataset. It does seem that large FSI is called for, in the form of both  $\delta$  and  $\sigma$  being sizable, but  $|\sigma - \delta| \lesssim 50^\circ$  to suppress  $K^-K^+$  mode.



**Fig. 3.** (a)  $A_{\pi\pi}$  and (b)  $S_{\pi\pi}$  vs.  $\sigma$

The difference between Fits 1 and 2 are marginal, except for  $S_{\pi\pi}$  becoming sizably negative as one moves from  $\phi_3 \sim 90^\circ$ , towards CKM fit result of  $60^\circ$ . However, the latter gives too small a  $\pi^-\pi^0$  rate (by  $3\sigma$ ) and somewhat unreasonably small form factor values.

It is remarkable that in the last two years, the discovery of sizable color suppressed  $D^0h^0$  modes, the appearance of  $A_{\text{CP}}(K^-\pi^+) < 0$ , evidence for  $\pi^0\pi^0$  rate  $> 10^{-6}$ , and the emerging (but unfortunately unsettled) picture for  $A_{\pi\pi}$  and  $S_{\pi\pi}$ , all suggest large FSI together with large  $\phi_3$  may be realized. We find  $\delta \sim 60^\circ$  and  $\sigma \sim 100^\circ$  on top of  $\phi_3 \sim 90^\circ$ . To test these, one needs refined measurement of  $\pi^0\pi^0$  rate, as well as finding  $K^-\bar{K}^0 > 10^{-6}$  but  $K^-K^+$  very suppressed. The value of  $A_{\text{CP}}(K^-\pi^0) < 0$  should be tested, as well as  $A_{\pi\pi} > 0$ . The  $S_{\pi\pi}$  parameter would be a good test for  $\phi_3$ .

If the two SU(3) rescattering phases differences  $\delta$  and  $\sigma$  bear out in the future, it would be a challenge to strong interaction physics to understand the origin of such large strong phases. In addition, large  $\sigma$  phase implies sizable “double annihilation” of initial state flavor content, which is a further mystery. We have only taken the utilitarian approach of putting these phases in as parameters. It is amusing that strong resistance to such strong phases come from not only the QCD factorization camp, but also from Regge approach camp. Neither seem willing to admit FSI phases larger than  $20^\circ$  or so.

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